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Adaptive Histogram Equalization Using Variable Regions.

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Abstract

Some recently introduced variations of the Adaptive Histogram Equalization (AHE) reduce the sometimes excessive noise amplification produced by AHE in regions whose pixels show only a small range of grey-values. In this contribution, we propose a different approach to this problem, by replacing each regional histogram that covers only a small range of grey-values by a linear combination with the histograms of neighbouring regions, instead of averaging it with a uniform distribution. The method is called Variable Region Adaptive Histogram Equalization (VRAHE).

1. Introduction

The advent of Picture Archiving and Communication Systems (PACS) has stimulated as well as facilitated the use of image enhancement techniques. In the near future PACS will provide an efficient means of image display and communication, particularly in hospitals [1].

As for communication, the availability of diagnostic images is largely improved, e.g., simultaneously at various, even remote locations. As for display, the diagnostic value of available images can often be improved by applying various image enhancement techniques on them. Enhancement is made relatively easy, as the images in a PACS are already available in digital form.

The enhancement technique of choice is the so-called histogram equalization (HE). In this technique, the *cumulative* histogram H of the grey-values g is used as the essential part of the function $F(g)$ that maps the original grey-values onto the transformed ones:

$$F(g) = g'_{min} + \Delta g' \frac{H(g)}{N}, \quad \text{with :} \quad \Delta g' \equiv g'_{max} - g'_{min},$$

in which g'_{max} and g'_{min} indicate the upper and lower limits of the transformed grey-values, respectively, and N represents the number of pixels over which the histogram has been taken [2]. If $H(g)$ is considered as a (piecewise) differentiable function, the *derivative* of this mapping function is directly proportional to the histogram itself:

$$h(g) \equiv \frac{dH(g)}{dg} \quad \Rightarrow \quad \frac{dF(g)}{dg} = \Delta g' \frac{h(g)}{N}.$$

The *global* histogram equalization is made *adaptive* (GHE → AHE) by taking the histogram over a contextual region (CR) instead of over the whole image:

$$F_{CR}(g) = g'_{min} + \Delta g' \frac{H_{CR}(g)}{N_{CR}}.$$

In many applications, the CR's cover the whole image as a mosaic. Although overlap between the CR's appears to have a positive influence on the quality of the result, it definitely has a negative influence on the required computing time. For that reason, in many cases the local $F(x, y, g)$ is computed by – e.g. bilinear – interpolation between 4 (tabulated) functions $F_{CR}(g)$ determined for the 4 nearest mosaic CR's. For a wide range of medical images, empirical estimates of the optimum number of CR's in an image are in the range between 4^2 and 16^2 [3,4].

Recently, an interesting and promising variation of AHE has been proposed, the so-called contrast limited or clipped version (CLAHE) [4]. In this technique, grey-values of the pixels in a CR that occur more frequently than a certain factor, S, times the average frequency of the global histogram, $\overline{h} \equiv N/\Delta g$ with $\Delta g \equiv g_{max} - g_{min}$, are "clipped". This means that the number of grey-values occurring in excess of $S\overline{h}$, is redistributed over the histogram in a uniform fashion. In CLAHE, the derivative of the mapping function is limited to:

$$\max(\frac{dF(g)}{dg}) = \Delta g' \frac{\overline{h}}{N} S = \frac{\Delta g'}{\Delta g} S.$$

Because of this limitation the small grey-value variations – in most cases caused by noise – in CR's with predominantly uniform grey-value cannot be over-amplified. This limited noise amplification eliminates the main disadvantage of the conventional AHE when applied to images containing CR's whose pixels show only a small range of grey-values, small compared to the global range Δg.

In practice, the values of the clipping factor S are limited to $1 \leq S \leq h^*_{max}$, with $h^*_{max} \equiv \max(h(g))/\overline{h}$. With $S = 1$, at its lower limit, $F_{CR}(g)$ becomes linear, as if the local histogram $h_{CR}(g)$ had already been equalized. Then its output simply is directly proportional with the original grey-values again. At its upper limit, with $S = h^*_{max}$ the contrast is not limited any more. The output of the mapping function then becomes identical with that of the conventional AHE. In between these extremes, the value of S controls the weighting of the average between a uniform distribution and the original histogram on input, and between the original and the equalized histogram on output.

A severe restriction to the use of histogram equalization in general is the requirement that the equalized histogram will have no empty bins. Assuming a bin width of 1 for both g and g', the requirement can only be fulfilled by putting:

$$\max(\frac{dF(g)}{dg}) \leq 1 \quad \text{or :} \quad \Delta g' \leq \frac{\Delta g}{S}.$$

This reduction in output grey-value resolution is only partly relieved in CLAHE by limiting the value of S. Otherwise, using GHE or regular AHE, S should be replaced by its maximum h^*_{max}:

$$h^*_{max} \equiv \frac{\max(h(g))}{\overline{h}} \equiv \frac{\max(h(g))\Delta g}{N} \quad \Rightarrow \quad \Delta g' \leq \frac{N}{\max(h(g))}.$$

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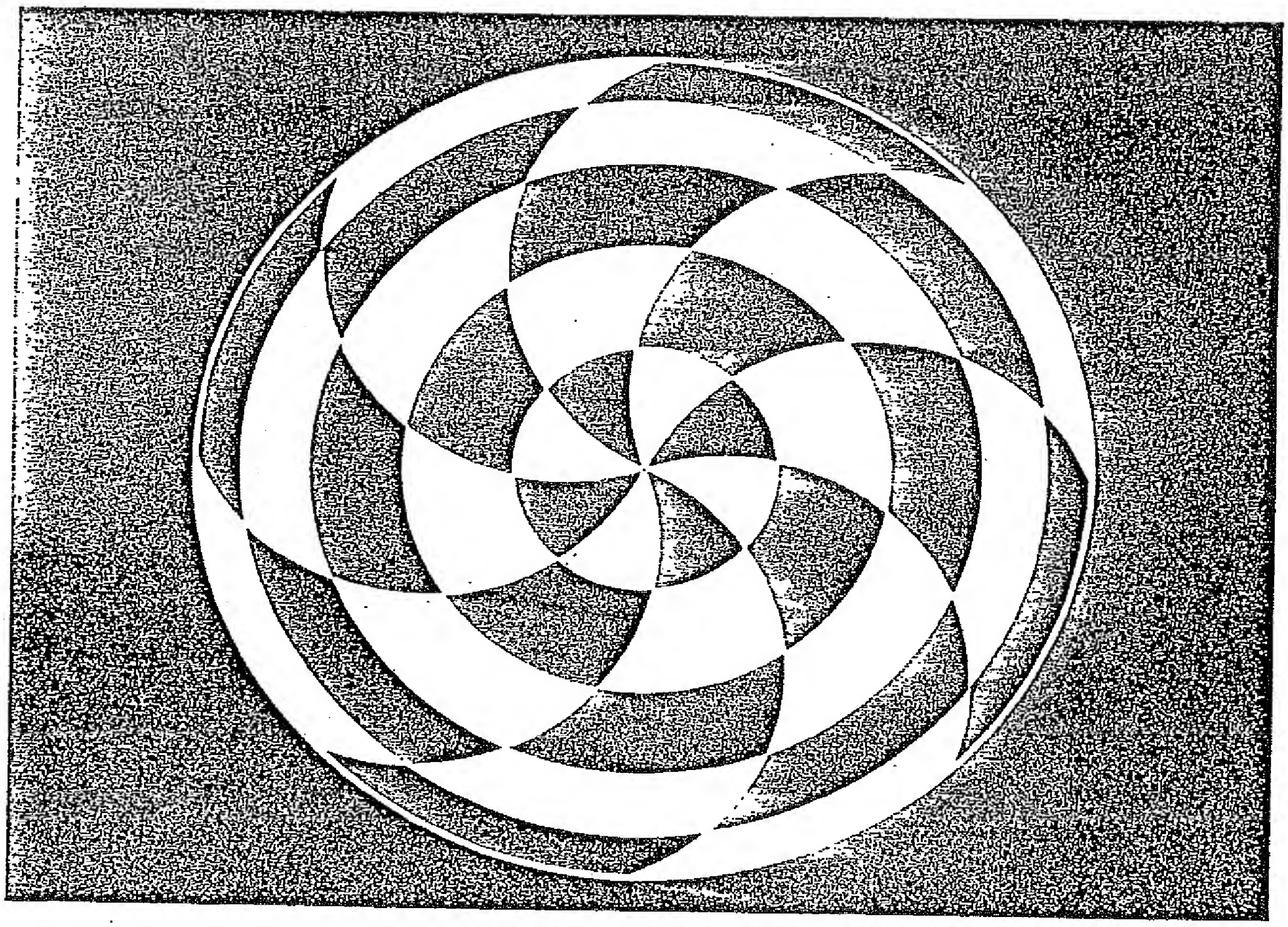

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Especially if the number of possible grey-values on input is limited by coarse digitization, both in AHE and in CLAHE problems arise with assigning these few grey-values to a smaller, or at most equal number of output values.

To overcome these problems, it has been suggested to add uniformly distributed random noise with an amplitude $\delta g \equiv (g_{i+1} - g_i)/2$ to each g_i before taking $F(g_i)$. By doing so, a single very frequently occurring input grey-value will be uniformly distributed over several output ones [5]. Essentially, this method is equivalent to considering the histogram as a probability density function (*pdf*) shaped like a bar-chart. Integrating such a *pdf* results in a monotonically increasing, piecewise linear $H(g)$. Although the method produces virtually flat output *histograms*, the random noise it introduces in the output *image* is not considered acceptable.

In this contribution, we propose a different approach to prevent excessive noise amplification in CR's with large value of h^*_{max}, i.e., a small value of $\max(\Delta g')$. Whenever the histogram of a CR shows such a pronounced peak, such that HE would lead to a very coarse digitization of the grey-values, that histogram is replaced by a linear combination of the histograms of one or more neighbouring CR's. This process is repeated until no CR histogram shows a value of $\max(\Delta g')$ below a distinct threshold any more. The procedure has been devised by analogy with the one employed in adaptive *thresholding*, in which histograms for which no local threshold can be computed are replaced by those of their neighbouring "CR's" [3].

2. Method

Three variations of the proposed method are presented, differing only in the algorithm adopted to replace the histogram of a contextual region (CR) with $\max_{CR}(\Delta g') <$ threshold T iteratively, until $\max_{CR}(\Delta g') \geq T$ for all CR's. Note that the threshold T essentially indicates the minimum number of output grey-values required.

The first variation is based on quadtrees:

- *Step 1:* Subdivide the image into a mosaic of 16×16, 8×8 or 4×4 CR's.
- *Step 2:* For each CR, compute the histogram $h_{CR}(g)$.
- *Step 3:* For each histogram, determine $\max_{CR}(\Delta g') = \lfloor N_{CR} / \max(h_{CR}(g)) \rfloor$.
- *Step 4a:* *(Quadtree algorithm)* Replace the histogram of every CR having $\max_{CR}(\Delta g') < T$, if any, by the average histogram of the CR's with $\max_{CR}(\Delta g') \geq T$, if any, having the same parent node in the quadtree. If no such CR is found, repeat step 4 with the histograms of one node higher in the quadtree, until action is no longer required, or the root node of the quadtree has been attained.
- *Step 5:* Find $F(g, x, y)$ by bilinear interpolation between the 4 nearest CR centers, as in [4].

Instead of replacing the histogram by that of CR's having the same parent node in the quadtree, one may also replace it by that of the "best" (four-connected) neighbour. Then only step 4 of the algorithm will be different:

- *Step 4b:* *(Best neighbour algorithm)* Start with the CR having $\min(\max_{CR}(\Delta g')) < T$, *and* at least one neighbour CR with $\max_{CR}(\Delta g') \geq T$, if any, and replace the histogram of the former CR by that of the latter with the highest value of $\max_{CR}(\Delta g')$. Repeat step 4, until action is no longer required. T_{max} is the same here as in step 4a.

A variation of this algorithm is to replace the histogram of the CR by the average of its (four-connected) neighbours, 2 if the CR is in a corner of the image, 3 if it is on its boundary, or 4 in the remaining cases. Only the "good" neighbours should be involved in the averaging, because otherwise the algorithm may not converge. Again only step 4 of the algorithm will be different:

- *Step 4c:* *(Average good neighbour algorithm)* Start with the CR having the largest $\max_{CR}(\Delta g') < T$, if any, and replace the histogram of this CR by the average histogram of those neighbour CR's that have $\max_{CR}(\Delta g') \geq T$. Repeat steps 3 and 4, until action is no longer required. In this case also, T_{max} is the same as in step 4a.

Note that there is, of course, a practical upper limit to T: $T_{max} = \max(\max_{CR}(\Delta g'))$. In other words, at least one of the CR's in the image should have $\max_{CR}(\Delta g') \geq T$. That CR then may act as a "seed", its histogram being propagated over all CR's in the image, regardless of which replacement algorithm is used in step 4.

3. Results

The VRAHE method has been implemented using Borland Turbo Pascal version 3 under MS-DOS on an IBM PC-AT personal computer with a Professional Graphics Adapter (PGA). The PGA can display 256 colours simultaneously from a palette of 4096 $(= 2^{12})$. Although this seems a sufficient number, it allows the display of only 16 shades of grey: every shade of grey consists of a mixture of red, green and blue in *equal* intensities only. For each of these colours, only 2^4 intensities are available, hence the palette of $(2^4)^3$ colours.

The results of the described algorithms, including CLAHE, are shown in figure 1 for a digitized image of a bone-marrow metaphase (first row), and for a transsectional CT-image of the abdomen (second row).

The leftmost photographs show the "original" images, only enhanced by contrast-stretching. The 256×256 images, with $\max(\Delta g') = 4$, have been subdivided into 16×16 square CR's of 16×16 pixels each. The other photographs in each row show the same images after VRAHE with $T = 16$, using steps 4a, 4b and 4c, and after CLAHE, respectively. In the upper row CLAHE has been applied with $S = 3$, in the lower row with $S = 8$, the highest values of S that resulted in $T = 16$ for both original images. The result of GHE is not shown here, because it is quite poor, with only $T_{max} = 4$ grey-values, of which 2 are used just for the background, as could be expected.

4. Discussion

The main differences among the three variations of the VRAHE method presented here are in the speed of convergence of the replacement algorithm, and in the maximum value of T that still guarantees convergence, T_{max}. The *quadtree* algorithm (employing step 4a) shows rapid convergence, in at most $\log_2 k$ steps if the image has been subdivided in $k \times k$ contextual regions (CR's).

The *best neighbour* algorithm (employing step 4b) shows slower convergence, because finding the *best* replacement candidate CR requires examination of all 4 neighbour CR's of each candidate CR. The advantage of step 4b over step 4a is that the histogram is replaced by that of its *closest* neighbour with a "good" histogram.

The *average good neighbour* algorithm (employing step 4c) has the advantage that histograms from a direct and often symmetrical neighbourhood are used in the replacing, with no more influence from distant

CR's than strictly necessary. However, convergence may be even slower than with the previous algorithm.

As mentioned already, the presentation of the VRAHE variations here is by no means comprehensive. Many other variations can be devised, e.g., by only partially replacing the original histogram of the CR, taking a weighted average between the original histogram and the one replacing it. These variations suffer from the same disadvantages as the ones employing steps 4b or 4c, and in many cases even to a greater extent: Slow convergence to a sub-optimal T_{max} that sometimes cannot even be computed beforehand.

As for the number of CR's the image is subdivided, mostly $k \times k$, the quadtree approach has the disadvantage that k must be an integer power of 2. This may present some problems if the image consists of sparsely distributed objects with small size on a background with practically uniform grey-level, as in the first "results" example. Then, e.g., 8×8 CR's may lead to an inacceptably low value of T_{max}, whereas 16×16 CR's may lead to inacceptably small CR's, depending on the image size. In those cases, step 4a should be replaced by step 4b, which allows to choose the region size accurately according to the size of the objects.

The time requirements of the VRAHE variations presented here will not differ substantially from those of any other AHE algorithm. The only difference is in the additional time requirement of the replacement algorithm. This is very low when compared with the time requirements for interpolation.

Note that for any AHE algorithm the time requirements cannot be reduced by using an almost purely integer implementation of the bilinear interpolation algorithm (based on runlength), in contrast with adaptive thresholding. The difference is that the 4 grey-values of the CR centers on the corners of a rectangle, in which a value for each pixel is interpolated, are no longer constants, but have to be selected according to the pixel's grey-value. Assuming the CR centers positioned on a rectangular grid, still some speed can be gained by tabulating the position-dependent coefficients for each pixel position on any rectangle between 4 CR centers as corners.

As for the evaluation of the results, it should be noted that this essentially requires subjective judgement by a panel of diagnostic experts of a series of images showing one or more features of diagnostic importance that are difficult to detect. This would eventually require an analysis of Receiver Operating Characteristic (ROC-) curves for the diagnostic problem at hand. However, such a ROC-analysis is considered beyond the scope of this study. Even if it was performed, its results would still be valid only for the diagnostic problem at hand. Apart from the enhanced detectability of specific diagnostic features, it can be stated that replacing a local histogram by a linear combination of *real* histograms from its vicinity is to be preferred over replacing it by a linear combination with an "abstract" *flat* histogram.

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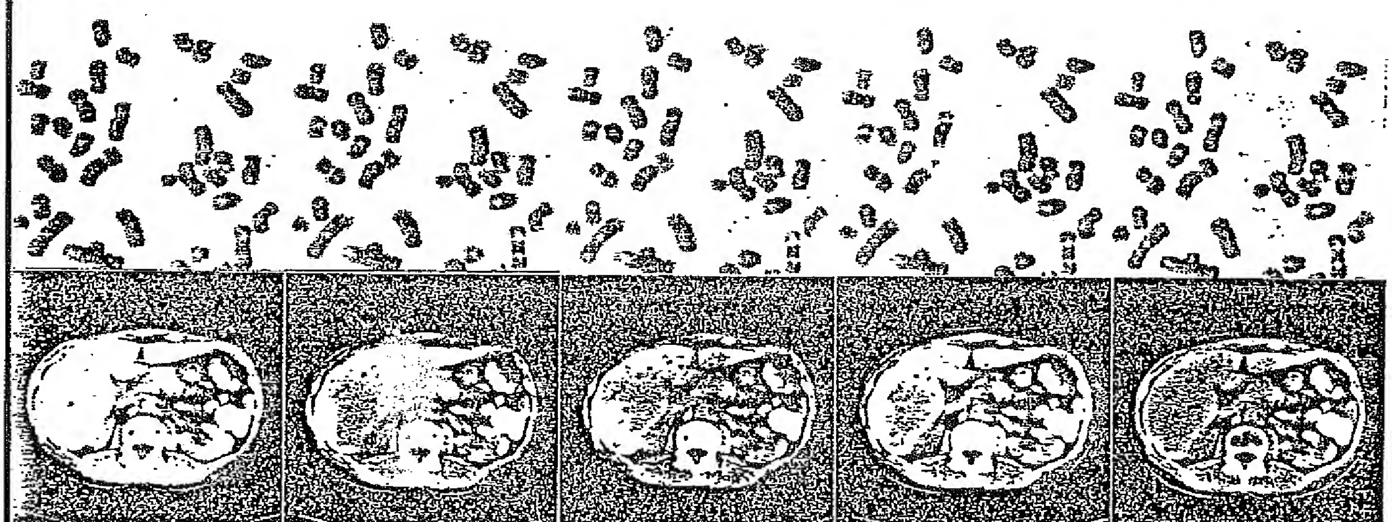

Figure 1: A digitized image of a bone-marrow metaphase (first row), and a transsectional CT-image of the abdomen (second row) The leftmost photographs show the "original" 256×256 images, only enhanced by contrast-stretching. The other photographs in each row show the same images after VRAHE with $T = 16$, using steps 4a, 4b and 4c, and after CLAHE, respectively.